New generation Λ hypernuclear spectroscopy with the (π^+, K^+) reaction by S-2S

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In this proposed experiment, we aim to prove Λ-hypernuclear spectroscopy with the energy resolution of 1 MeV (FWHM), which is the best among reaction spectroscopy with hadron beams, by using S-2S. The new spectrometer S-2S is expected to lead to the high resolution in a resulting missing mass. Solid targets of $^{\mathrm{nat.}}$Li, ^{10}B, and $^{\mathrm{nat.}}$C with the thickness of 1 g/cm^2 will be used for the measurements of $^{7}_{\Lambda}$Li, $^{10}_{\Lambda}$B, and $^{12}_{\Lambda}$C, respectively. The good resolution allows us to calibrate the energy by using Λ binding energies of $^{7}_{\Lambda}$Li$(1/2^+, 5/2^+)$ which are reliable data in the past emulsion and γ-ray experiments. The new calibration by

using ${}^{7}_{\Lambda}\mathrm{Li}(1/2^+, 5/2^+)$ will lead to a 100-keV accuracy, and will be a game changer. This proposed experiment has a great impact because the Λ binding energy of ${}^{12}_{\Lambda}\mathrm{C}^{\mathrm{g.s.}}$, which is the energy reference for all of previous (π^+, K^+) data, will be determined with the accuracy of 100 keV. This is the first experimental attempt of the accurate measurement on binding energy of ${}^{12}_{\Lambda}\mathrm{C}^{\mathrm{g.s.}}$ by a counter experiment. In addition, by comparing ${}^{10}_{\Lambda}\mathrm{B}^{\mathrm{g.s.}}$ with its mirror nucleus ${}^{10}_{\Lambda}\mathrm{Be}^{\mathrm{g.s.}}$, an effect of the charge symmetry breaking (CSB) in the mass number of 10 (A = 10) is able to be investigated thanks to such a high accuracy. This experiment is the first step of the new generation (π^+, K^+) experiment, foreseeing future studies of other hypernuclei such as systems with the larger mass number.

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EXECUTIVE SUMMARY

The (π^+, K^+) reaction is used in the experiment. Momentum vectors of π^+ and K^+ at the reaction point are measured by the K1.8-beam line and S-2S, respectively, to perform missing-mass spectroscopy of Λ hypernuclei. The π^+ beam at $p_\pi = 1.05$ GeV/c is planned to be exposed to experimental targets, $^{\mathrm{nat.}}$Li, ^{10}B, and $^{\mathrm{nat.}}$C, with the areal density of 1 g/cm^2. The central momentum setting for S-2S, which measures the K^+ momentum, is 0.72 GeV/c. The requested beam time is 10.5 days (252 hours) in total with the assumption of 5×10^6 pions/spill at the spill cycle of 5.2 seconds, including beam-through runs. The experimental setup is the same as that for E70, in which a Ξ hypernucleus is investigated via the (K^-, K^+) reaction, except for central momentum settings of the spectrometers and the beam polarity.

I. INTRODUCTION

Spectroscopic data of hypernuclei are used for a study of the hyperon-nucleon (YN) and hyperon-hyperon (YY) interactions. The most straightforward way to investigate an interaction between baryons is a scattering experiment. However, it is not easy to perform for hyperons because of their short lifetimes of the order of 100 ps. While there exist some data of the hyperon-nucleon scattering, the statistics are very limited, as well as the measured energy ranges and angular acceptances [1]. Most of those data sets were obtained with hydrogen bubble chambers for various reaction channels. Recently, a remarkable scattering experiment for the $\Sigma^{\pm}$p channels with much higher statistics was realized at K1.8 beam line in the J-PARC E40 Experiment [2–4]. New type of scattering data between the Λ and nucleon were also reported from the CLAS Collaboration at Jefferson Lab (JLab) [5]. Furthermore, a new proposal (P86) [7] and LoI [6] were submitted to J-PARC PAC for new measurements on the Λp scattering. Thus, new generation of hyperon-nucleon scattering data with the improved statistics will be soon available, and they are promising to improve our understanding of the two-body YN interaction in the near future.

The data of hyperon scattering experiments are limited yet so far. At the moment, therefore, the spectroscopic data of the hypernuclei are vital for the studies of the YN and YY interactions. The hypernuclear spectroscopy by using hadron, electron, and heavy ion beams was developed and performed at various experimental facilities such as KEK, CERN, BNL, J-PARC, GSI, JLab etc. to investigate the YN and YY interactions. The energy resolution has been the key to reveal the structure of Λ hypernuclei. Hypernuclear γ-ray spectroscopy which achieved a few keV energy resolution is one of the best resolution tools. Very small splittings of the states due to spin-spin interaction, spin-orbit splitting, and tensor interaction were resolved with hyperball and hyperball-J detectors at KEK, BNL and J-PARC. The Λ single-particle orbit energies were observed at KEK with the SKS spectrometer with the missing-mass resolution of 1.45, 1.95, 1.65 and 2.4 MeV/c^2 (FWHM) for hypernuclear productions of ${}^{12}_{\Lambda}$C, ${}^{51}_{\Lambda}$V, ${}^{89}_{\Lambda}$Y, and ${}^{208}_{\Lambda}$Pb, respectively by using targets with the thickness of 0.86–2.82 g/cm^2 [8]. At JLab, better mass resolutions of 1.3, 1.1 0.8, 0.5 MeV/c^2 (FWHM) were achieved for hypernuclei of ${}^{7}_{\Lambda}$He, ${}^{9}_{\Lambda}$Li, ${}^{10}_{\Lambda}$B, and ${}^{12}_{\Lambda}$B, respectively by using targets with the thickness of 0.09–0.21 g/cm^2 [9].

The measured species of hypernuclei is only about 40 although large efforts have been devoted since the first event of a hypernucleus was discovered in nuclear emulsion plates in 1953. One finds that the experimental data of hypernuclei are very scarce by comparing with a fact that the more than 3000 species were experimentally identified for so-called normal nuclei. In addition, data qualities such as precision and accuracy in their binding energies were limited. In order to expand the number of species and to improve the data qualities for the hypernuclear spectroscopy, experimental efforts have been being devoted at the state-of-the-art accelerator facilities all around the world. One of the attempts which are being prepared to carry out is the missing-mass spectroscopy with the $(e, e'K^+)$ reaction at JLab. The experiment at JLab has achieved the best precision in reaction spectroscopy to date, which is 0.5 MeV/c^2 (FWHM) [10]. The success of the $(e, e'K^+)$ reaction spectroscopy at JLab was led by an introduction of High Resolution Kaon Spectrometer (HKS) [11, 12]. The new spectrometer HKS has the momentum resolution of $\Delta p/p = 2 \times 10^{-4}$ in FWHM, maintaining a reasonable solid-angle acceptance of about 7 msr. The hypernuclear project at JLab is ongoing, and next attempt is to measure Λ binding energies of $^{3}_{\Lambda}$H, $^{4}_{\Lambda}$H, $^{40}_{\Lambda}$K, $^{48}_{\Lambda}$K, and $^{208}_{\Lambda}$Tl [13–15]. The expected mass resolution and energy accuracy for the next hypernuclear experiment at JLab are 0.5–1.0 MeV/c^2 (FWHM) and $<$ 100 keV, respectively. The high accuracy measurement is possible thanks to an energy calibration by using elementary productions of $p(e, e'K^+)\Lambda$,Σ^0. The masses of Λ and Σ^0 are well known with the accuracy of a few to a few 10 keV. The uncertainty which originates from the energy references (the masses of Λ and Σ^0) are negligibly small compared to other systematic errors. The hypernuclear spectroscopy with such a high accuracy is expected to give us important information on the isospin-dependent three-body ΛNN interaction in medium to heavy mass systems. As for the investigations of the few body systems, the high accuracy measurements would be fundamental information for constructing the ΛN interaction model. The next hypernuclear experiment for the mass numbers of $A = 3$–208 at JLab is planned to be performed in 2025 at JLab Hall C.

The experiments at J-PARC and JLab are complementary because the reactions, which they use, produce different hypernuclei even from the same nuclear target. For example, the use of the different reaction enables us to investigate mirror hypernuclei. Binding-energy comparisons between mirror hypernuclei particularly for the light mass systems provide information on the charge symmetry breaking (CSB) in the ΛN interaction. Promotion of hypernuclear studies in both facilities is a key to enhance the effectiveness for investigating the ΛN interaction.

II. GOAL OF THE PROPOSED EXPERIMENT

The goal of the present experiment is to prove the feasibility of high-precision and high-accuracy spectroscopy of Λ hypernuclei by means of the (π^+, K^+) reaction with S-2S. The goal resolution and accuracy are 1 MeV (FWHM) and 0.1 MeV, respectively, which would be the best among existing (π^+, K^+) data. The experiment uses the experimental targets of $^{\mathrm{nat.}}$Li, ^{10}B, and $^{\mathrm{nat.}}$C, producing $^{7}_{\Lambda}$Li, $^{10}_{\Lambda}$B, and $^{12}_{\Lambda}$C, respectively. This proposed experiment has aims other than the proof of experimental feasibility. It is argued that the known Λ-binding energy of $^{12}_{\Lambda}$C, which is the calibration source for all of existing data of (π^+, K^+) spectroscopy, needs to be corrected by about 0.5 MeV. The present experiment directly confirms the 0.5-MeV shift. In addition, the CSB is investigated by comparing the binding energy of $^{10}_{\Lambda}$B to that of $^{10}_{\Lambda}$Be.

A. $^{12}\mathbf{C}(\pi^+, K^+)^{12}_{\Lambda}\mathbf{C}$

The Λ binding energy of $^{12}_{\Lambda}$C, $B_{\Lambda}^{\mathrm{emul.}}(^{12}_{\Lambda}\mathrm{C}^{\mathrm{g.s.}})$, was measured to be 10.37 MeV by the emulsion experiment [16]. However, the need of the 0.54-MeV correction for the binding energy was suggested in Ref. [17]. An independent study by FINUDA collaboration consistently suggested that the 0.6-MeV correction is necessary [18]. The need of energy correction has a great impact because it had been used for the energy calibration in the past (π^+, K^+) experiments, and it shows that the most of existing Λ hpernuclear data are necessary to be corrected. What we aim to do in the proposed experiment is to accurately determine the $B_{\Lambda}(^{12}_{\Lambda}\mathrm{C}^{\mathrm{g.s.}})$ to directly confirm the energy shift.

The accurate data of $B_{\Lambda}(^{12}_{\Lambda}\mathrm{C}^{\mathrm{g.s.}})$, that we aim to provide in the present experiment, will be the important reference to be used as the energy calibration for future experiments. In the present experiment, $B_{\Lambda}(^{7}_{\Lambda}\mathrm{Li})$ is planned to be used. However, as shown in Sec. IV B 2, the use of $B_{\Lambda}(^{7}_{\Lambda}\mathrm{Li}; 1/2^+, 5/2^+)$ requires a high energy resolution because the energy separation between reference peaks is only a few MeV. On the other hand, $B_{\Lambda}(^{12}_{\Lambda}\mathrm{C})$ is much easier to use for the calibration because the reference peak is well separated from the other prominent peaks (i.e. the *s*-shell peak has a ∼10 MeV difference from the *p*-shell peak). Therefore, the peak of $B_{\Lambda}(^{12}_{\Lambda}\mathrm{C})$ could be used even for an experiment in which a high resolution is not required. It is needless to say that the ^{12}C target is much easier to buy and handle as well, compared to the Li target.

B. $^{10}\mathbf{B}(\pi^+, K^+)^{10}_{\Lambda}\mathbf{B}$

Fig. 1 shows a comparison between binding-energy spectra obtained at KEK [19] and JLab [17] in which (π^+, K^+) and $(e, e'K^+)$ reaction were used for the same ^{10}B target. The (π^+, K^+) reaction converts a neutron into a Λ. In contrast, the $(e, e'K^+)$ does convert a proton into Λ. Therefore, these reactions can produce mirror hypernuclei from the same target. The major structures, peak number #1–4, are very similar, and they agree well with the theoretical predictions. However,

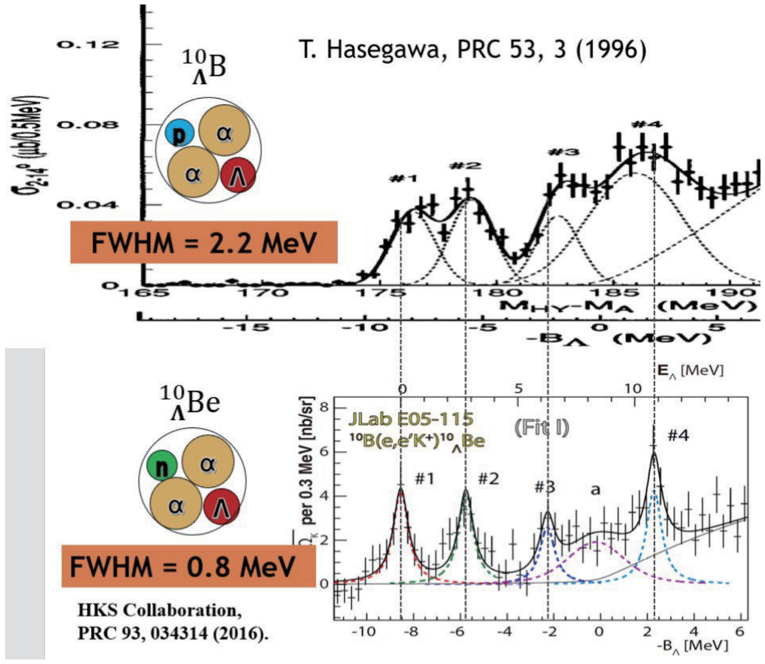

FIG. 1. Comparison of binding-energy spectra between $^{10}_{\Lambda}$B [19] and $^{10}_{\Lambda}$Be [17] measured by experiments with (π^+, K^+) and $(e, e'K^+)$ reactions, respectively.

in $^{10}_{\Lambda}$Be spectrum obtained at JLab, some events which are distributed widely between peak #3 and #4 were successfully observed thanks to the better energy resolution. States between #3 and #4 were not predicted before the experiment. Recently, A. Umeya et al. tried to reproduce the high resolution spectrum, and found that the model space of their shell-model calculation is necessary to be extended [20]. Now, these events are considered to be corresponding to a Λ in p-shell couples in parallel to an α-α cluster in the core nucleus. It is a proof that the Λ could be a good probe to explore the cluster structure of core nucleus. A similar phenomenon was discussed for the spectroscopic data of $^{9}_{\Lambda}$Be measured at KEK [21]. Moreover, a recent result of a neutron rich Λ hypernucles $^{9}_{\Lambda}$Li, that was shown by the JLab's experiment, indicates that a cluster structure of ^{5}He and t is developed for a particular excited state of the core nucleus ^{8}Li [22]. High resolution spectroscopic data of Λ hypernuclei would reveal cluster structures or deformations of their core nuclei. The proposed experiment will lead to further investigations of the hypernuclear clusters/deformations in the future. As the first step of such a study, we aim to confirm the cluster structure of α-α in the core nucleus ^{9}B, which could not be observed in the old experimental data of $^{10}_{\Lambda}$B at KEK, by improving the energy resolution by a factor of more than two.

In addition, the CSB effect can be studied by comparing the ground-state binding energies of $^{10}_{\Lambda}$B with that of $^{10}_{\Lambda}$Be. The CSB in the p-shell hypernuclei is expected to be much smaller than

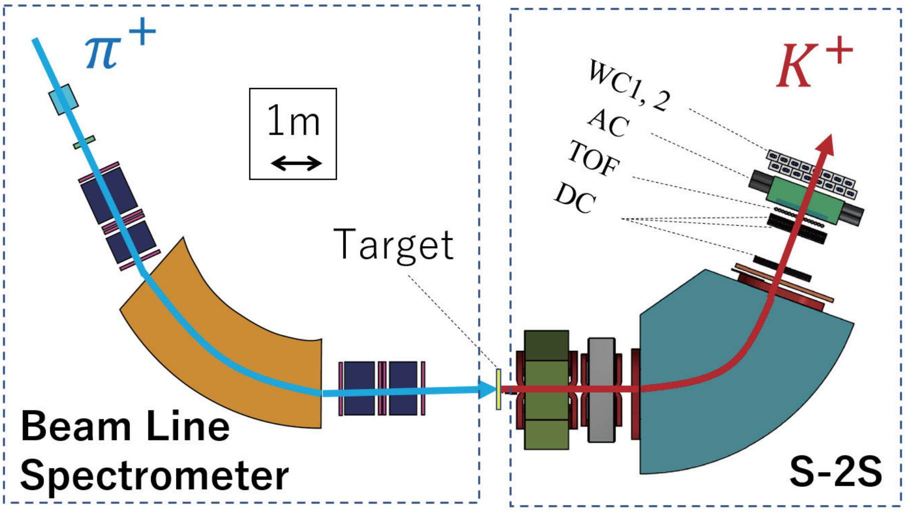

FIG. 2. Schematic of the experimental setup in which the (π^+, K^+) reaction will be used for hypernuclear production. π^+ beams and K^+'s from the (π^+, K^+) reaction will be detected by the K1.8 beam-line spectrometer and S-2S, respectively. Momentum vectors of π^+ and K^+ which will be used to reconstruct the hypernuclear mass (missing mass) will be obtained by analyses of the beam-line spectrometer and S-2S, respectively

that of the *s*-shell system as predicted by A. Gal et al. [23] and E. Hiyama et al. [24]. We aim to provide the binding energy of ${}^{10}_{\Lambda}$B with the accuracy of 0.1 MeV which is 5–10 times better than that of previous (π^+, K^+) experiments, and the new data would allow us to investigate the CSB effect. It is worth noting that the goal accuracy is comparable to that of its mirror nucleus ${}^{10}_{\Lambda}$Be that was measured at JLab.

III. EXPERIMENTAL SETUP

The experimental setup is the same as that of J-PARC E70 experiment [25] except for the experimental target and the magnetic field settings for the spectrometers. In this section, we describe the differences to be noted.

A. Experimental Method

The (π^+, K^+) reaction will be used for the hypernuclear production. π^+ beams which will be momentum-analyzed by the K1.8 beam line spectrometer will be delivered on a 1-g/cm^2 ^{12}C target, and K^+'s will be measured by S-2S. The target will be uninstalled when beam-through runs are carried out. Figure 2 shows a schematic of the experimental setup. Once the momentum vectors of π^+ and K^+ are obtained by the spectrometers, the missing mass (M_H, hypernuclear mass) will be calculated as follows:

$$\begin{aligned} M_\mathrm{H} &= \sqrt{E_\mathrm{H}^2 - (\vec{p_\mathrm{H}})^2} \\ &= \sqrt{(E_\pi + M_\mathrm{T} - E_\mathrm{K})^2 - (\vec{p_\pi} - \vec{p_K})^2} \end{aligned} \tag{1}$$

where $E_{\mathrm{H},\pi,K}$ and $p_{\mathrm{H},\pi,K}$ are the energies and the momenta of a hypernucleus, π^+, and K^+. M_T is the nuclear mass of a target. The Λ binding energy B_Λ will be derived as the following:

$$B_\Lambda = M_\mathrm{core} + M_\Lambda - M_\mathrm{H} \tag{2}$$

where $M_{\mathrm{core},\Lambda}$ are the masses of a core nucleus and a Λ.

B. Kinematics

The beam momentum of 1.05 GeV/c where the production cross section of Λ from a neutron is large will be employed as was used in the previous hypernuclear experiments at KEK. The differential cross sections of the elementary process $n(\pi^+, K^+)\Lambda$ are 780, 400 and 250 μb/sr for $p_b =$ 1050, 1200 and 1500 MeV/c, respectively at the K^+ scattering angle of $\theta_{\pi K} = 0°$ in the theoretical calculation [26]. Table I shows the K^+ momenta for the ${}^{12}\mathrm{C}(\pi^+, K^+){}^{12}_{\Lambda}\mathrm{C}$ and ${}^{89}\mathrm{Y}(\pi^+, K^+){}^{89}_{\Lambda}\mathrm{Y}$ reactions at the beam momenta of $p_b =$ 1050, 1200 and 1500 MeV/c. Survival ratios of K^+ in the K^+ spectrometer S-2S that has a path length of about 8.6 m from a target to the most downstream detector for the central ray are also summarized in Tab. I

TABLE I. The calculated momenta of p_K from the (π^+, K^+) reaction assuming the different beam momenta $p_\mathrm{b} = 1050, 1200$ and 1500 MeV/c. The K^+ scattering angle of $\theta_{\pi K} = 0°$ was assumed.

Reaction	p_b [/(MeV/c)] π^+	p_s [/(MeV/c)] K^+	K^+ survival ratio
${}^{12}\mathrm{C}(\pi^+, K^+){}^{12}_{\Lambda}\mathrm{C}$ (g.s.)	1050	717	0.20
	1200	893	0.28
	1500	1223	0.39
${}^{89}\mathrm{Y}(\pi^+, K^+){}^{89}_{\Lambda}\mathrm{Y}$ (g.s.)	1050	746	0.22
	1200	919	0.29
	1500	1248	0.40

The momentum transferred to a Λ by the $n(\pi^+, K^+)\Lambda$ reaction is 400–450 MeV/c at $p_\pi =$ 1050 MeV/c for a range of K^+ scattering angle of 0°–10°. On the other hand, the momentum transferred is reduced to be 300–400 MeV/c at $p_\pi = 1500$ MeV/c for the same angle range, and one can expect that a sticking probability of Λ in a nucleus is larger although the elementary production cross section is smaller. In addition, at $p_\pi = 1500$ MeV/c, a survival ratio of K^+ against its decay is also larger by a factor of 1.8 (= 0.40/0.22; see Tab. I) compared with the case of $p_\pi = 1050$ MeV/c. Therefore, there may be a room to consider the usage of higher beam momentum. In terms of the yield, for example, $p_\pi = 1500$ MeV/c would be better to use if the production cross section at $p_\pi = 1500$ MeV/c is larger than about a half of the cross section at $p_\pi = 1050$ MeV/c. However, it should be noted that the energy resolution in a resulting spectrum gets worse as the beam momentum is larger as shown in Sec. IV A.

C. Particle Detectors in S-2S

S-2S has five drift chambers for a particle tracking (SDC1,2,3,4,5), a plastic scintillation detector for a time-of-flight (TOF) measurement, and two types of Cherenkov detectors for a K^+ identification. Radiation media of the Cherenkov detectors are aerogel and pure water which have the refractive indices of $n = 1.05$ and 1.33, respectively. Major background particles will be π^+'s and protons in S-2S, and the aerogel Cherenkov detector (AC) yields signals when π^+'s pass through. In addition, π^+'s and K^+'s will be detected by the water Cherenkov detector (WC). Therefore, we will be able to identify K^+'s by applying a condition of $\mathrm{TOF} \otimes \mathrm{WC} \otimes \overline{\mathrm{AC}}$ at both online (trigger) and offline (analysis) stages. Here, TOF stands for a hit condition of the plastic scintillation detector. The above condition of KID is the same as that for E70. The Cherenkov detectors were designed for the J-PARC E70 experiment in which the K^+ identification will be done at the central momentum of $p^{\mathrm{cent.}}_{\mathrm{S-2S}} = 1.37$ GeV/c which is about two times higher than that of the proposed experiment. WC will have signals from the Cherenov radiations by protons in addition to K^+'s and π^+'s in the E70 experiment because of the higher momentum setting, and the protons needs to be rejected by applying a pulse height selection online and offline. On the other hand, the proton rejection will be easier in the proposed experiment because WC will have no signals or much smaller for protons. It is noted that AC does not sensitive to K^+'s and protons in the E70 experiment as well as the proposed experiment. Refer to Ref. [27] for details about the K^+ identification in S-2S. Major differences from the E70 experiment that has been approved to be on the Stage 2 are the beam momentum and its polarity. The proposed experiment needs positive charged beams whereas negative charged beams are going to be used in the E70 experiment.

IV. EXPECTED RESULT

A. Energy resolution

The binding energy resolution was estimated by a simple Monte Carlo simulation. The method to obtain the Λ binding energy from the missing-mass measurement is described in Sec. III. In the MC simulation, angle resolutions $\Delta x'$ ($x' \equiv \frac{p_x}{p_z}$) and $\Delta y'$ ($y' \equiv \frac{p_y}{p_z}$) were both assumed to be $\sigma = 3$ mrad which is a conservative value. Assumed momentum resolutions were $\Delta p_{\mathrm{s}}/p_{\mathrm{s}} = 6 \times 10^{-4}$ (FWHM) and $\Delta p_{\mathrm{b}}/p_{\mathrm{b}} = 10 \times 10^{-4}$ (FWHM) for S-2S [28, 29] and the beam line spectrometer, respectively. The beam momentum resolution of $\Delta p_{\mathrm{b}}/p_{\mathrm{b}} = 10 \times 10^{-4}$ (FWHM) has been proven in the past hypernuclear experiments with Superconducting Kaon Spectrometer (SKS). The momentum straggling in a target was simulated by a Geant4 MC simulation for the areal density of 1 g/cm^2. The momentum straggling was found to be $\Delta p_{\mathrm{strag.}} \simeq 350$ keV/c when the particles passed through the full length of the target. Positions of the hypernuclear production will be random within the target length (volume in reality), and an average of the straggling effect is estimated to be $\Delta p^{\mathrm{ave.}}_{\mathrm{strag.}} = \frac{\Delta p_{\mathrm{strag.}}}{2}$ for each of π^+ and K^+. The momentum straggling effects for π^+ and K^+ in the target were taken into account for the energy resolution estimation as follows:

$$\Delta p'_{\pi,K} = \sqrt{\Delta p^2_{\mathrm{b,s}} + (\Delta p^{\mathrm{ave.}}_{\mathrm{strag.}})^2}. \tag{3}$$

The estimated energy resolution was found to be about 1260 keV (FWHM) for the target thickness of 1 g/cm^2. Figure 3 shows the simulation result. No significant differences in the energy resolution between hypernuclear productions from the carbon and yttrium targets were found if the target areal density and beam conditions were the same.

The energy resolution was also evaluated in the case of $\Delta p_{\mathrm{b}}/p_{\mathrm{b}} = 5 \times 10^{-4}$ (FWHM) which is the design value for the beam line spectrometer whereas the other parameters were fixed as original ones. It was found that the energy resolution becomes 860 keV (FWHM) if the beam line spectrometer has the resolution of $\Delta p_{\mathrm{b}}/p_{\mathrm{b}} = 5 \times 10^{-4}$ (FWHM). In addition, if the areal density of the target was reduced to 0.1 g/cm^2 which leads to a much less momentum straggling effect (the amount of a few 10 keV), the expected resolution became about 670 keV in FWHM. However, the usage of the 0.1-g/cm^2 target is not reasonable particularly for a heavier mass target with the beam condition that we assumed here because ten times longer beam time (e.g. a beam time of about 180 days is needed to have 30 events of ${}^{89}_{\Lambda}$Y ground state for which the differential cross section of about 0.5 μb/sr is assumed) is required than the case of the 1-g/cm^2 target.

The SKS has the momentum resolution $\Delta p_{\mathrm{s}}/p_{\mathrm{s}}$ of the order of 10^{-3}, and thus a calibration of the

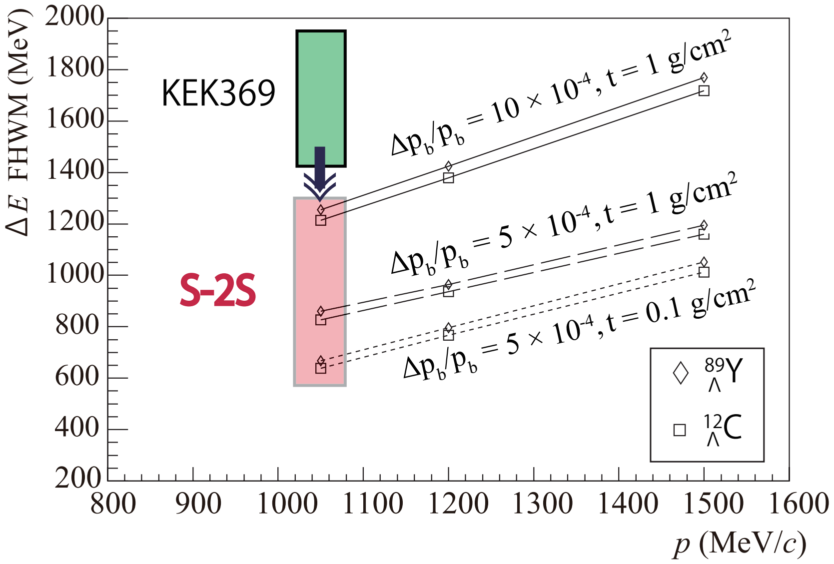

FIG. 3. The Λ binding energy resolution as a function of beam momentum for the proposed experiment that will use the (π^+, K^+) reaction. The energy resolution was evaluated by a simple MC simulation in which angle resolutions ($\Delta x'$ and $\Delta y'$ for both of the beam line spectrometer and S-2S) and a momentum resolution of S-2S were fixed to be 3 mrad and $\Delta p_{\mathrm{s}}/p_{\mathrm{s}} = 6 \times 10^{-4}$ FWHM. The momentum straggling effect which was estimated by a Geant4 MC simulation was taken into account.

K1.8 beam-line spectrometer to achieve the design resolution was not necessary and has not been seriously done. It may be possible to achieve the design resolution of the beam-line spectrometer by using calibration data taken with S-2S that has the momentum resolution of $\Delta p_{\mathrm{s}}/p_{\mathrm{s}} = 6 \times 10^{-4}$ (FWHM).

To evaluate contributions of the momentum and angular resolutions to the missing-mass resolution, the derivatives of the missing mass with the momenta ($p_{\pi,K}$) and $\theta_{\pi K}$ were calculated, and the derivatives were multiplied by the momentum and angular resolutions. The calculation result is shown in Table II. We assumed the two cases of the beam momentum resolutions that include the momentum straggling effect as described in Eq. (3). One is $\Delta p'_{\pi(1)}$ for $\Delta p_{\mathrm{b}}/p_{\mathrm{b}} = 5 \times 10^{-4}$ (FWHM) and the other is $\Delta p'_{\pi(2)}$ for $\Delta p_{\mathrm{b}}/p_{\mathrm{b}} = 10 \times 10^{-4}$ (FWHM). The contribution from the beam momentum is larger than that of the K^+ momentum to the missing mass resolution. The contribution from the angular resolution is zero because $\theta_{\pi K} = 0°$ was assumed in the calculation. It is noted that the angle-resolution contribution is still negligible small even if $\theta_{\pi K}$ has a finite value within the S-2S acceptance.

To summarise, we could achieve the binding energy resolution of 860–1260 keV in FWHM assuming the resolution of the beam-line spectromter is $\Delta p_{\mathrm{b}}/p_{\mathrm{b}} = (5\text{–}10) \times 10^{-4}$ (FWHM) and the

TABLE II. Contributions of the momentum and angular resolutions to the missing mass resolution with an assumption of the $^{12}\mathrm{C}(\pi^+, K^+)^{12}_{\Lambda}\mathrm{C}$ reaction at $p_\pi = 1050$ MeV/c. The momentum straggling effect which comes from a 1-g/cm^2 target was taken into account as described in Eq. (3). The beam momentum resolutions assumed for the calculation ($\Delta p_\mathrm{b}/p_\mathrm{b}$ in FWHM) are 5×10^{-4} and 10×10^{-4} for $\Delta p'_{\pi(1)}$ and $\Delta p'_{\pi(2)}$, respectively.

V	$\frac{\partial M_\mathrm{H}}{\partial \mathrm{V}}$ $[/(\frac{\mathrm{MeV}}{c^2}/\mathrm{MeV})]$ or $[/(\frac{\mathrm{MeV}}{c^2}/\mathrm{rad})]$	ΔV (/MeV) or (/rad)	$\lvert\frac{\partial M_\mathrm{H}}{\partial \mathrm{V}} \times \Delta \mathrm{V}\rvert$ $[/(\frac{\mathrm{MeV}}{c^2})]$
p_π	0.962	$\Delta p'_{\pi(1)} = 0.525$	0.505
		$\Delta p'_{\pi(2)} = 1.05$	1.01
p_K	−0.980	$\Delta p'_K = 0.430$	0.421
$\theta_{\pi K}$	0	$\Delta\theta_{\pi K}$	0

target thickness is 1 g/cm^2. The resolution of $\Delta p_\mathrm{b}/p_\mathrm{b} = 10 \times 10^{-4}$ (FWHM) has been proven in the past hypernuclear experiment with SKS. However, we may be able to calibrate it to achieve the designed momentum resolution by using the new spectrometer S-2S that has the compatible resolution with that of the design value of the beam-line spectrometer. In addition to the momentum resolution of the beam-line spectrometer, the momentum resolution of S-2S may be different from what we expect due to, for example, the beam-optics effect of hypernuclear productions from the finite volume of target *etc.* It is vital to understand the momentum resolutions for the beam-line spectrometer and S-2S in order to extract physics information for some cases in coming experiments. The proposed experiment will provide data to develop the calibration method to achieve the energy resolution as close as the designed specifications, and will be a good benchmark of S-2S performance for a wide range of the momentum setting.

B. Expected accuracy

We aim to achieve the high momentum resolution $\Delta p/p$ of the order of 10^{-4} in the proposed experiment. The calibration method to obtain such a high energy resolution is not trivial. For example, in hypernuclear experiments at JLab in which the $(e, e'K^+)$ reaction was used, great efforts were devoted to a development of the energy calibration method that led to the momentum resolution of $\Delta p/p = 2 \times 10^{-4}$ (FWHM) [9]. In the hypernuclear experiment at JLab, backward transfer matrices that had elements up to the 6th order for both a scattered electron and a K^+ spectrometers were used to reconstruct momentum vectors at a production point for the missing mass spectroscopy. When data analysis started by using initial backward transfer matrices that were made by Geant4 MC simulation, the momentum resolution $\Delta p/p$ was found to be only the order of 10^{-3}. The worse resolution came from a difference between magnetic fields of the real experiment and the simulation by which the initial matrices were made. A semi-automatic calibrator for the backward transfer matrices by using MINUIT algorithm was carefully developed considering an absolute energy scale and a linearity for whole acceptance. The calibration method led to not only a good accuracy but also the goal momentum resolution of $\Delta p/p = 2 \times 10^{-4}$ in FWHM, and the best resolution and accuracy were successfully achieved among counter experiments of hypernuclei [10]. We may apply a similar technique to the beam-optics calibration of S-2S combined with the K1.8 beam-line spectrometer. Another possible way to realize the goal momentum resolution would be an introduction of the machine learning (ML) technique as studied in Ref. [30].

In both cases of the above calibration methods, the number of events used for the calibration should be at least as large number as the parameters to be optimized. The summed number of parameters for a description of the momenta of the K1.8 beam-line spectrometer and S-2S could be the order of 10^2–10^3 depending on their mathematical descriptions that are being considered.

1. Calibration source

In the proposed experiment, we are going to derive the momenta by backward transfer matrices or MLs for the missing mass reconstruction. On the other hand, particle angles at the production point are going to be measured by tracking devices around the production target. We propose to use two types of data for the momentum calibration: (i) beam through events (with/without a target and various momentum settings for both the beam line spectrometer and S-2S), and (ii) the hypernuclear events of ${}^{7}_{\Lambda}$Li. Table III shows settings of the central momenta for the K1.8 beam-line spectrometer and S-2S for the beam through runs.

TABLE III. Central momenta of the beam-line spectrometer and S-2S for the beam-through runs. It is noted that physics runs will run with the central momenta of 1050 and 740 MeV/c for the beam-line spectrometer and S-2S, respectively.

	K1.8 Beam-line Spectrometer	S-2S
	Central-momentum setting [/(MeV/c)]	
Beam Through Data	650, 685, 720, 755, 790	720
	1050	950, 1000, 1050, 1100, 1150

2. Absolute energy calibration

The beam-through data will be used as a constraint of momentum consistency between the beam-line spectrometer and S-2S as follows, $\vec{p_{\rm b}} = \vec{p_{\rm S}}$. However, the consistency between the beam line and S-2S does not ensure the absolute energy in a resulting binding-energy spectrum. In order to calibrate the absolute energy, the binding energies of $B_\Lambda({}^{7}_{\Lambda}\mathrm{Li}; 1/2^+, 5/2^+)$ which are well known are planned to be used.

The energy calibration by using the ${}^{7}_{\Lambda}$Li hypernucleus has not been done in the previous missing-mass experiment with the (π^+, K^+) reaction due to the insufficient energy resolution. Figure 4 shows the missing-mass spectrum obtained in the previous experiment at KEK PS. The energy resolution of 1.81 MeV (FWHM) was not able to separate the first and second peaks which are labeled as #1 and #2, respectively in Fig. 4. The (π^+, K^+) reaction at the forward scattering angle has a small probability to flip the spin of a nucleon when the nucleon is converted into Λ. Therefore, the peaks of #1 and #2 are considered to be mainly composed of the states of $J^\pi = 1/2^+$ (ground state) and $5/2^+$, respectively. The γ-ray energy of E2 transition $(5/2^+ \rightarrow 1/2^+)$ was measured by the γ-ray spectroscopy with the Germanium detector [31, 32], and thus, the energy separation between them is well known. However, the unseparated structure due to the insufficient resolution in the missing-mass spectrum did not allow one to use the ${}^{7}_{\Lambda}$Li energy as the energy-calibration reference. Figure 5 shows the expected binding-energy spectrum of ${}^{7}_{\Lambda}$Li in the proposed experiment

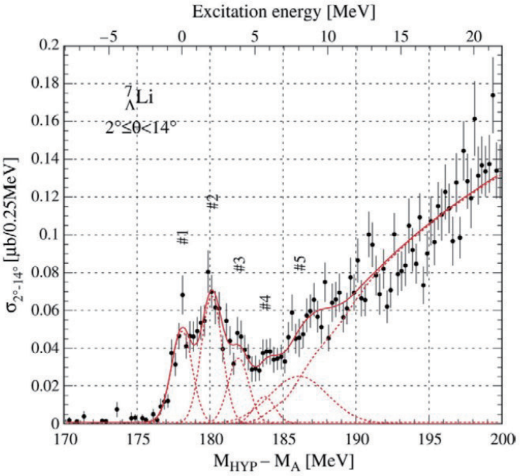

FIG. 4. Spectrum of the reaction cross section as a function of the excitation energy obtained at KEK PS for the ^{7}Li$(\pi^+, K^+){}^{7}_{\Lambda}$Li reaction [21]. The energy resolution was 1.81 MeV in FWHM.

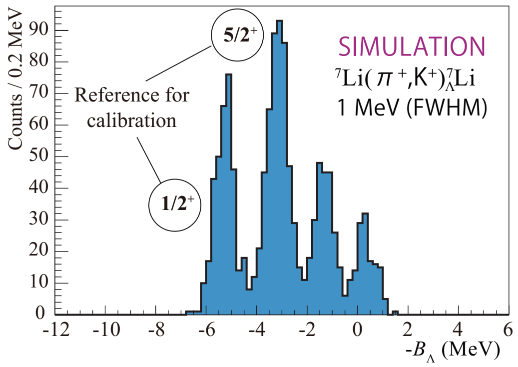

FIG. 5. Expected binding-energy spectrum of ^{7}Li$(\pi^+, K^+){}^{7}_{\Lambda}$Li with 3 days of beam time (72 hours) on the 1 g/cm^2-thick target in the proposed experiment. The assumed energy resolution is 1 MeV in FWHM.

TABLE IV. Reference energies ($B_\Lambda^{\rm ref.}$) for the present experiment. Expected statistical errors on the binding-energy measurement ($|\Delta B_\Lambda^{\rm stat.}|$) in the present experiment are shown in the last column.

Hypernucleus	J^π	$B_\Lambda^{\rm ref.}$ (/MeV)	Remarks	$\|\Delta B_\Lambda^{\rm stat.}\|$ (/MeV)
${}^{7}_{\Lambda}$Li	$1/2^+$ (g.s.)	5.58 ± 0.03	Ref. [16]	< 0.03
	$5/2^+$	3.53 ± 0.03	Refs. [16, 31, 32]	< 0.03

from the 1-g/cm^2 thick ^{7}Li target. The $1/2^+$ and $5/2^+$ states are clearly separated thanks to the better energy resolution. Therefore, these well known binding energies $B_\Lambda({}^{7}_{\Lambda}\mathrm{Li}; 1/2^+, 5/2^+)$ are usable for the absolute energy calibration for the first time. Table. IV shows the uncertainty of the calibration sources. The ground state energy of ${}^{7}_{\Lambda}$Li was measured by the emulsion experiment with relatively higher statistics compared to the other hypernuclei. For these energy references, the statistical error is about 30 keV, and the systematic error is about 40–50 keV. On the other hand, the statistical error of the calibration data in the present experiment is about 30 keV. Therefore, the error originating from the calibration is about 65 keV.

Other factors which could contribute to the binding energy uncertainty is expected to come from an energy-loss correction in materials and the linearity of the energy scale. The uncertainty of the energy loss correction would be minimized by using data of beam-through runs with and without the experimental target, and data of ${}^{12}_{\Lambda}$C measurements with different target thickness. The uncertainty from the energy-loss correction is expected to be about a few 10 keV to 50 keV that is mainly originated from the statistical errors on the calibration peaks such as ${}^{7}_{\Lambda}$Li, beam-through, and ${}^{12}_{\Lambda}$C from the thick target. We aim to minimize the systematic error from the energy linearity down to about 50 keV or less, which was found to be achievable for the case of a similar spectrometer HKS. In the present experiment, the total uncertainty including statistical and systematic errors is evaluated to be about 100 keV.

C. Yield

The yield $N_{\rm H}$ was estimated by using the following equation:

$$N_{\rm H} = \frac{d\sigma}{d\Omega} \times \Delta\Omega \times \epsilon \times N_{\rm target} \times N_{\rm beam} \tag{4}$$

where $\frac{d\sigma}{d\Omega}$, $\Delta\Omega$, $N_{\rm target}$ and $N_{\rm beam}$ are the differential cross section, the solid angle acceptance of S-2S, the number of target nucleus with the unit of cm^{-2}, and the number of incident beam on target. The ϵ is a total efficiency that takes into account the K^+ survival ratio of 0.2 (see Table 3) and various factors such as DAQ, detectors and analysis efficiencies. Here, we took $\epsilon = 0.1$ ($= 0.2 \times 0.5$). Table V summarized parameters used for the yield estimation. The beam intensity was assumed to be 5×10^6 pions per spill with the spill cycle is 5.2 seconds. The expected yield per hour per μb/sr per g/cm^2 for various hypernuclear productions are obtained as shown Tab. VI. The expected number of events per day is about 110 counts for the ground-state productions of ${}^{7}_{\Lambda}$Li, ${}^{10}_{\Lambda}$B, and ${}^{12}_{\Lambda}$C. The assumed differential cross sections are 3, 4 and 5 μb/sr, respectively which are based on the previous measurements [19].

TABLE V. Estimated yields of the ground states of ${}^{7}_{\Lambda}$Li, ${}^{10}_{\Lambda}$B, and ${}^{12}_{\Lambda}$C. Assumed parameters for the yield estimation are also shown.

Hypernucleus	${}^{7}_{\Lambda}$Li (g.s.)	${}^{10}_{\Lambda}$B (g.s.)	${}^{12}_{\Lambda}$C (g.s.)
Differential Cross Section $\frac{d\sigma}{d\Omega}$ [/(μb/sr)]	3	4	5
Target (thickness)	^{7}Li (1 g/cm^2)	^{10}B (1 g/cm^2)	^{12}C (1 g/cm^2)
The Number of Target N_{target} (/cm^{-2})	8.60×10^{22}	6.02×10^{22}	5.02×10^{22}
Solid Angle Acceptance $\Delta\Omega$ (/msr)	55		
Total Efficiency ϵ	0.1		
Beam Intensity	5M pions / spill (5.2 sec)		
Beam time (/hours)	72	72	72
Yield	352	330	345

TABLE VI. Expected yield of hypernuclei per hour per μb/sr per g/cm^2. The beam intensity of 5M pions per spill with the spill cycle of 5.2 seconds was assumed.

Hypernuclei	${}^{7}_{\Lambda}$Li	${}^{10}_{\Lambda}$B	${}^{12}_{\Lambda}$C	${}^{40}_{\Lambda}$Ca	${}^{51}_{\Lambda}$V	${}^{89}_{\Lambda}$Y	${}^{209}_{\Lambda}$Pb
Yield per (μb/sr) per (g/cm^2) per DAY	39.3	27.5	22.9	6.89	5.40	3.09	1.32

D. Expected Spectrum

Figure 6 shows the expected binding-energy spectrum from the 1-g/cm^2 thick ^{12}C target. The experiment aims to determine the ground state energy ($J^\pi = 1^-$) which corresponds to the first peak of the figure (labeled as s_Λ). The statistical error on the binding energy determination is about 30 keV or less.

Figure 7 shows the expected binding-energy spectrum from the 1-g/cm^2 thick ^{10}B target in the proposed experiment. The statistical error for the determination of the peak positions is about

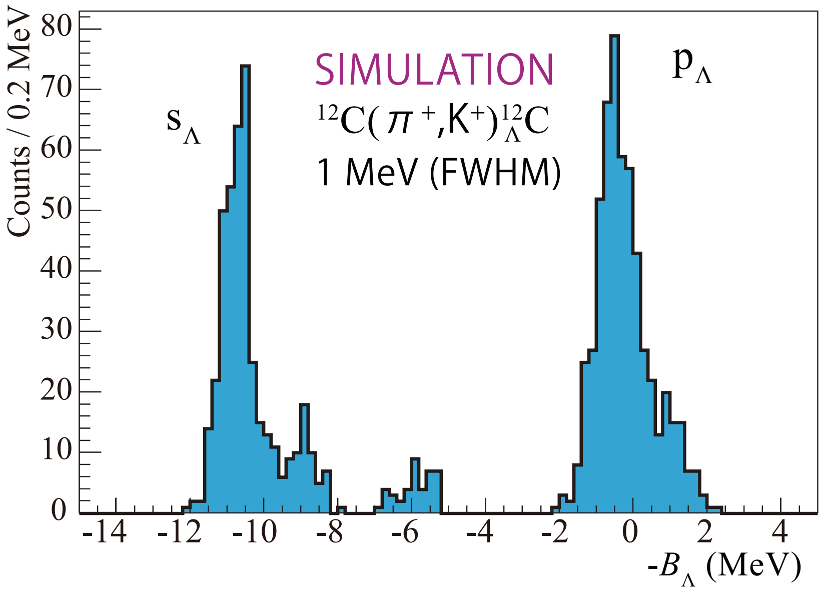

FIG. 6. Expected spectrum of ^{12}C$(\pi^+, K^+)^{12}_{\Lambda}$C with 3 days of beam time (= 72 hours) on the 1 g/cm^2-thick target. The assumed energy resolution is 1 MeV in FWHM.

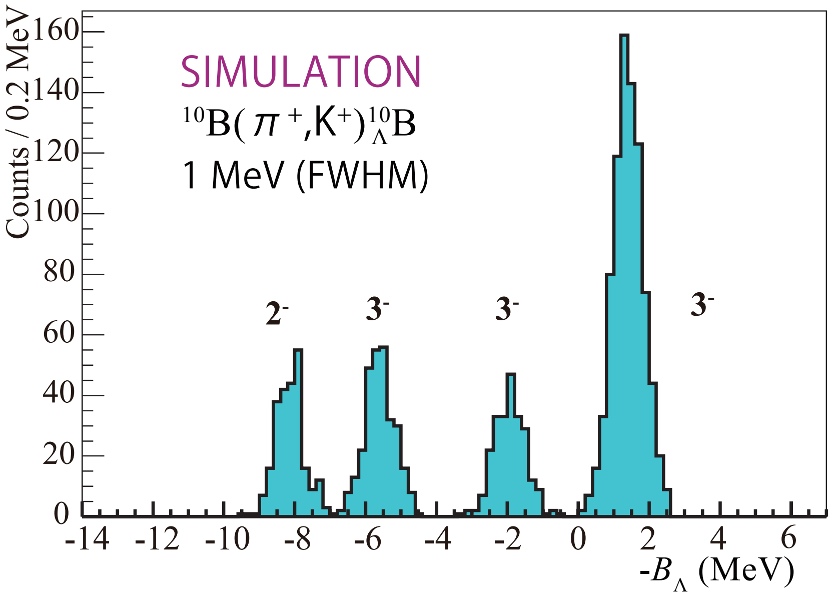

FIG. 7. Expected spectrum of ^{10}B$(\pi^+, K^+)^{10}_{\Lambda}$B with 3 days of beam time (= 72 hours) on the 1 g/cm^2-thick target. The assumed energy resolution is 1 MeV in FWHM.

30 keV. However, an additional systematic error needs to be considered when the ground state is determined. The first peak should correspond to the 2^- state because the (π^+, K^+) reaction has a small amplitude for the spin flip at the forward angle. However, the ground state could be the 1^- state depending on the ΛN interaction. γ rays from the transition from 2^- to 1^- were not

observed in the past experiment. The non-detection of γ ray indicates that the separation between these states is less than 100 keV, or that the 2^- is the ground state. There is no problem for the determination of the ground state if it is the latter case. However, an additional error needs to be considered in the case that the 1^- state is the ground state. Fortunately, the production cross section of the 2^+ state is much larger for the (π^+, K^+) reaction, and separation could be only 100 keV at maximum. Thus, the possible systematic error originating from the fact that 1^- could be the ground state is estimated to be much less than 100 keV, which is likely to be only a few 10 keV.

V. BEAM-TIME REQUEST

We request a total of 252 hours of beam time as shown in Tab. VII. The beam-time request is based on an assumption of the beam intensity of 5×10^6 pions per spill with the spill cycle of 5.2 seconds (0.96M pions per second) with no downtime of the accelerator.

TABLE VII. Beam-time request in the proposed experiment. We assumed the beam intensity of 5M pions per spill with the spill cycle of 5.2 seconds. The momentum sets of S-2S and the beam-line spectrometer at $(p^{\rm cent.}_{\rm Beam}, p^{\rm cent.}_{\rm S-2S}) = (1050, 720)$ MeV/c is planned to be used.

Target (thickness [/(g/cm^2)])	Beam time (/hours)	Remarks
^{12}C (0,1,3)	12	Beam-through runs (see also Tab. III)
^{7}Li (1)	72	$^{7}_{\Lambda}$Li for the energy calibration (Fig. 5)
^{10}B (1)	72	$^{10}_{\Lambda}$B production (Fig. 7)
^{12}C (1)	72	$^{12}_{\Lambda}$C production (Fig. 6)
^{12}C (3)	24	To study the energy-loss and straggling effects (ΔE_Λ gets worse by a factor of about 1.5–2.0)
Total	**252**	-

VI. SUMMARY

We aim to prove Λ-hypernuclear spectroscopy with the energy resolution of 1 MeV (FWHM), which is the best among reaction spectroscopy with hadron beams. Solid targets of $^{\text{nat.}}$Li, ^{10}B, and $^{\text{nat.}}$C with the thickness of 1 g/cm^2 will be used for the measurements of $^{7}_{\Lambda}$Li, $^{10}_{\Lambda}$B, and $^{12}_{\Lambda}$C, respectively. The new calibration by using $^{7}_{\Lambda}\mathrm{Li}(1/2^+, 5/2^+)$ will lead to an accuracy of 100 keV for the energy determination. This is the first accurate measurement on binding energy of $^{12}_{\Lambda}\mathrm{C}^{\mathrm{g.s.}}$ by a counter experiment. In addition, by comparing $^{10}_{\Lambda}\mathrm{B}^{\mathrm{g.s.}}$ with its mirror nucleus $^{10}_{\Lambda}\mathrm{Be}^{\mathrm{g.s.}}$, an effect of the charge symmetry breaking (CSB) in the A = 10 system will be investigated. The proposed experiment is the first step of the new generation (π^+, K^+) experiment, foreseeing future studies of other hypernuclei such as heavier mass systems.

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